Demonstration of Unconditional One-Way Quantum Computations for Continuous Variables

Ryuji Ukai,[1] Noriaki Iwata,[1] Yuji Shimokawa,[1] Seiji C. Armstrong,[1,2] Alberto Politi,[1,3] Jun-ichi Yoshikawa,[1] Peter van Loock,[4] and Akira Furusawa[1]

[1]*Department of Applied Physics, School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*
[2]*Centre for Quantum Computation and Communication Technology, Department of Quantum Science, Research School of Physics and Engineering, The Australian National University, Canberra ACT 0200 Australia*
[3]*Centre for Quantum Photonics, H. H. Wills Physics Laboratory & Department of Electrical and Electronic Engineering, University of Bristol, Merchant Venturers Building, Woodland Road, Bristol, BS8 1UB, United Kingdom*
[4]*Optical Quantum Information Theory Group, Max Planck Institute for the Science of Light, Institute of Theoretical Physics I, Universität Erlangen-Nürnberg, Staudtstr.7/B2, 91058 Erlangen, Germany*
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One-way quantum computation is a very promising candidate to fulfill the capabilities of quantum information processing. Here we demonstrate an important set of unitary operations for continuous variables using a linear cluster state of four entangled optical modes. These operations are performed in a fully measurement-controlled and completely unconditional fashion. We implement three different levels of squeezing operations and a Fourier transformation, all of which are accessible by selecting the correct quadrature measurement angles of the homodyne detections. Though not sufficient, these linear transformations are necessary for universal quantum computation.

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Quantum computing promises to exploit the laws of quantum mechanics for processing information in ways fundamentally different from today's classical computers, leading to unprecedented efficiency [1,2]. The cluster model of quantum computation (QC) is a recently proposed alternative to the conventional circuit model [3–8]. In this model, unitary operations are achieved indirectly through measurements on a highly entangled quantum state—the cluster state. Cluster computation is achieved through the following steps: (1) preparation of an entangled cluster state and an input state for processing, (2) entangling operation on these two states, (3) measurements on most subsystems of the cluster state and feed-forward of their outcomes, and (4) occurrence and readout of the output in the remaining unmeasured subsystems of the cluster. Universality, i.e., realization of arbitrary unitary operations is achieved by adjusting the measurement bases, sometimes also dependent on the results of earlier measurements [5,6].

Several experiments of one-way quantum computation have been reported for discrete-variable (qubit) systems using single photons [9–12]. These demonstrations of one-way quantum computation work in a probabilistic way, since the resource cluster is generated only when the photons that compose the cluster are produced and detected. Another typical feature of the single-photon-based cluster computation experiments is that the usual input states, $|+\rangle$, are prepared as part of the initial cluster states. These properties would pose severe limitations when unitary gates are to be deterministically applied online to an unknown input state which is prepared independently of the cluster state, for instance, as the output of a preceding computation.

In contrast, we report in this paper on unconditional one-way quantum computation experiments conducted on independently prepared input states. These inputs, as well as the entangled cluster state, are continuous-variable states. The price to pay for this is a set of stronger requirements on universality. Not only do we need at least one nonlinear element to achieve completely universal QC over continuous variables [7,13], we also have to cover all linear transformations, which, for a single optical mode, consist of arbitrary displacement, rotation, and squeezing operations in phase space. Our scheme represents the ultimate module for arbitrary linear transformations of arbitrary one-mode quantum optical states. It can be directly incorporated into a full, universal cluster-based QC together with a nonlinear element such as measurements based on photon counting [14] (for a discussion on the fidelity when concatenating our module using finitely squeezed cluster states, and on its scalability into a full, fault tolerant measurement-based QC, see supplemental material [15] and Refs. [16–18]).

We use a continuous-variable four-mode linear cluster state as a resource [8]. An approximate version of this cluster state can be obtained deterministically by combining four squeezed vacuum states on an 80%-transmittance beam splitter and two half beam splitters (HBSs) [19–21].

Recently, it was shown that the complete set of one-mode linear unitary Bogoliubov (LUBO) transformations, corresponding to Hamiltonians quadratic in $\hat{x}$ and $\hat{p}$, can be implemented using a four-mode linear cluster state as a resource [22]. The measurements required to achieve these operations are efficient homodyne detections with quadrature angles θ_i, which are easily controllable by adjusting the local oscillator phases in the homodyne detectors.

The total procedure then consists of the teleportation-based [23–25] coupling $\hat{M}_{\rm tele}(\theta_{\rm in}, \theta_1)$, followed by two elementary, measurement-based, one-mode operations $\hat{M}(\theta_i)$ [14,26,27] (see supplemental material [15]):

$$|\psi_{\rm out}\rangle = \hat{M}(\theta_3)\hat{M}(\theta_2)\hat{M}_{\rm tele}(\theta_{\rm in}, \theta_1)|\psi_{\rm in}\rangle. \quad (1)$$

Each step can be decomposed into three inner steps, namely, a ϕ rotation, squeezing, and a φ rotation in phase space: $\hat{R}(\varphi)\hat{S}(r)\hat{R}(\phi)$ with $\hat{R}(\theta) = e^{i\theta(\hat{x}^2+\hat{p}^2)}$ and $\hat{S}(r) = e^{ir(\hat{x}\hat{p}+\hat{p}\hat{x})}$ [28]. We have $\hat{M}_{\rm tele}(\theta_{\rm in}, \theta_1) = \hat{R}(-\theta_+/2)\hat{S}(r)\hat{R}(-\theta_+/2)$ with $r = \log\tan(\theta_-/2)$ and $\theta_\pm = \theta_{\rm in} \pm \theta_1$, while $\hat{M}(\theta_i) = \hat{R}(\phi_i)\hat{S}(r_i)\hat{R}(\phi_i)$ with $r_i = \log\frac{\sqrt{k_i^2+4}+k_i}{2}$, $\phi_i = \frac{\pi}{2} - \tan^{-1}\frac{\sqrt{k_i^2+4}-k_i}{2}$, and $k_i = 1/\tan\theta_i$.

In our experiment, we demonstrate four types of LUBO transformations: the Fourier transformation $\hat{F} = \hat{R}(\pi/2)$ (90° rotation); and three different x-squeezing operations $\hat{S}(r)$ with $r = \ln 10^{a/20}$, $a = 3, 6, 10$ [dB]. Figures 1(a) and 1(c) show the abstract illustration and the experimental setup, respectively. We employ the experimental techniques described in Refs. [19,29] for the generation of the cluster state and the feed-forward process, respectively.

The Fourier transformation is achieved by choosing for step (3) measurement quadrature angles $(\theta_{\rm in}, \theta_1, \theta_2, \theta_3)$ as $(90°, 0°, 90°, 90°)$, see [15].

The measurement results for the Fourier transformation of a coherent-state input are shown in Fig. 2. As clearly shown in Fig. 2(a), the input is a coherent state with amplitude 17.7 ± 0.2 dB. The output state is shown in Fig. 2(b). The peak level of trace Fig. 2a(i) is 17.5 ± 0.2 dB higher than the shot noise level (SNL), which is the same level as the input within the error bar. We acquire the peak of the input by measuring x, while we obtain the peak of the output by measuring p, corresponding to a 90° rotation in phase space. These measurement results confirm that the Fourier transformation is applied to the input coherent state.

The quality of the operation can be quantified by using the fidelity, defined as $F = \langle\Psi_{\rm ideal}|\hat{\rho}_{\rm out}|\Psi_{\rm ideal}\rangle$. In the specific case of our experiment, the fidelity for a coherent-state input as given above is $F = 2/\sqrt{(1+4\sigma_{\rm out}^x)(1+4\sigma_{\rm out}^p)}$, where $\sigma_{\rm out}^x$ and $\sigma_{\rm out}^p$ are the variances of the position and momentum operators in the output state, respectively [30]. We obtain $\sigma_{\rm out}^x = 2.9 \pm 0.2$ dB [Fig. 2b(iii)], and $\sigma_{\rm out}^p = 2.8 \pm 0.2$ dB (not shown) above the SNL with a vacuum input, corresponding to a fidelity of $F = 0.68 \pm 0.02$. This is in good agreement with the theoretical result $F = 0.71$, where an average squeezing level of -5.5 dB is taken into account.

Another fundamental element of the LUBO transformations is squeezing. A sequence of teleportation coupling $\hat{M}_{\rm tele}(\theta_{\rm in}, \theta_1)$ followed by elementary one-mode one-way

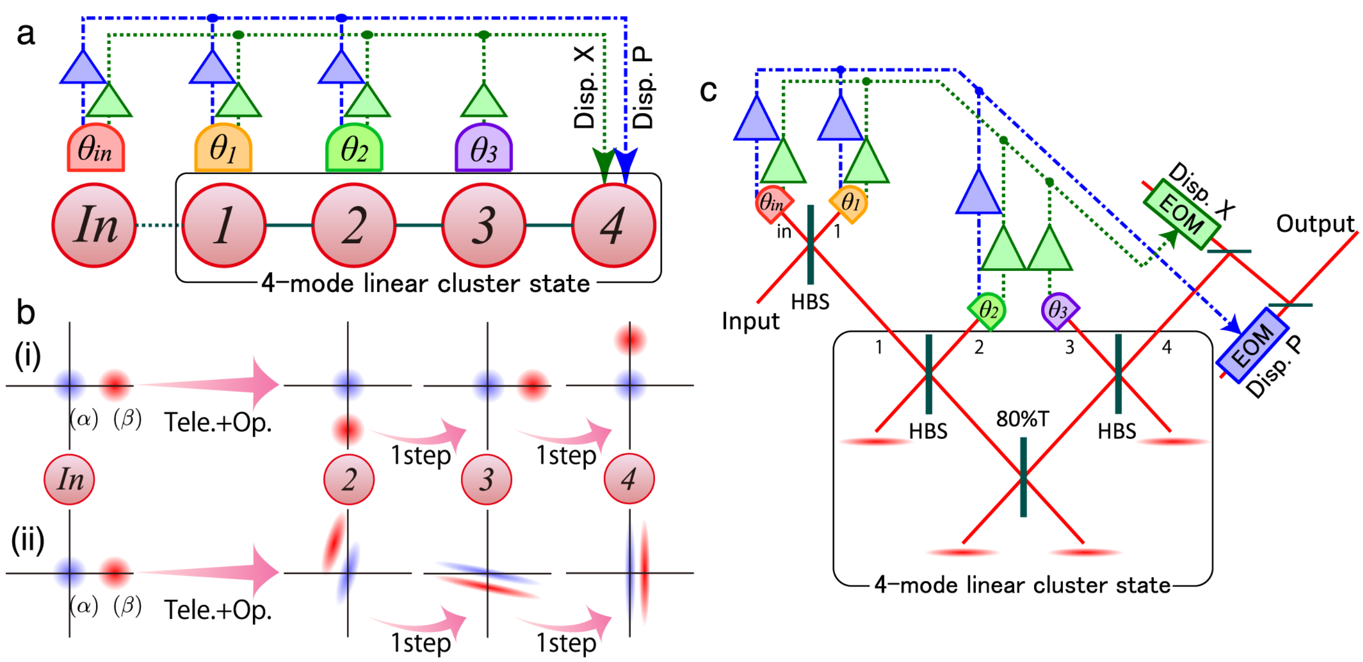

FIG. 1 (color online). (a) Abstract illustration and (c) experimental setup of one-mode LUBO transformations using a four-mode linear cluster state. There is a 1-to-1 correspondence between (a) and (c). Squeezed vacuum states are generated by subthreshold optical parametric oscillators containing periodically poled $KTiOPO_4$ crystals as nonlinear media. (b) Phase space representations of quantum states in each step of the Fourier transformation (i) and the 10 dB x-squeezing operation (ii), starting with a vacuum state input (α) and an x-coherent state input (β) Disp.: displacement in phase space, Tele.: teleportation, Op: operation, EOM: electro-optical modulator, HBS: half beam splitter.

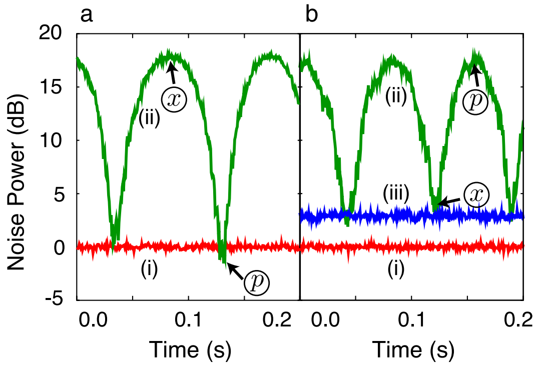

FIG. 2 (color online). Fourier transformation operation; (a) Measurement results of the input state. Trace (i) shows the shot noise level (SNL) and (ii) shows the phase scan of the input state. (b) Measurement results of the output state. Trace (i) shows the SNL, (ii) shows the phase scan of the output state, and (iii) shows the measurement result of the x quadrature with a vacuum input. The measurement quadrature angle is determined through the relative phase between the signal beam and the local oscillator beam. The measurement frequency is 1 MHz and the resolution and video bandwidths are 30 kHz and 300 Hz, respectively. Traces a(i), b(i), and b(iii) are averaged 20 times.

operations $\hat{M}(\theta_i)$ is required in order to extract squeezing without rotations [see Fig. 1(b)(ii)].

We implemented three different squeezing operations with three different sets of quadrature measurement angles $(\theta_{\text{in}}, \theta_1, \theta_2, \theta_3)$:

$$\begin{aligned} &(-42.5°, 62.4°, 63.5°, 76.0°),\\ &(-41.4°, 72.2°, 41.9°, 74.4°),\\ \text{and}\quad &(-47.7°, 79.2°, 25.9°, 78.4°), \end{aligned} \tag{2}$$

resulting in 3, 6, and 10 dB x-squeezing operations, respectively, (see supplemental material). In all these squeezing gates, the inputs are chosen to be coherent states with a nonzero amplitude in x (x-coherent) or in p (p-coherent), and these amplitudes are 14.7 dB $\pm$ 0.2 dB.

Figure 3(a) shows the measurement results of the 10 dB x-squeezing operation on the x-coherent state. In this figure, the extra dotted lines are plotted for comparison, in order to show the levels of the input state of x [Fig. 3a(v); 14.7 dB] and p [Fig. 3a(vi); SNL]. We obtain signal levels of 5.1 $\pm$ 0.2 dB and 11.5 $\pm$ 0.2 dB above the SNL for the measurement of the x and p quadratures of the output, respectively. The level of the x quadrature of the output [Fig. 3a(iii)] is about 10 dB lower than that of the input [Fig. 3a(v)], while the variance of the p quadrature of the output [Fig. 3a(iv)] increases by about 10 dB compared to that of the input [Fig. 3a(vi)]. These observations are consistent with a 10 dB x-squeezing operation. Note that the x and p quadratures of the output have additional noises. These are caused by the finite squeezing of the cluster state and would vanish in the limit of infinite cluster squeezing.

In order to show the nonclassical nature of the output state, we also use a vacuum state as the input [Fig. 3(b)]. The measured variance of the x quadrature is $-0.5 \pm$ 0.2 dB, which is below the SNL, thus confirming nonclassicality.

Finally, we demonstrate the controllability of the one-way quantum computations. Both theoretical curves (with -5.5 dB resources) and measured results for the three levels (3, 6, and 10 dB) of x squeezing are plotted in Fig. 3(c). Three kinds of input states are used here: a vacuum state; an x-coherent state; and a p-coherent state. As can be seen in Fig. 3(c), the measurement results agree well with the theoretical curves, and all the operations are indeed controlled by the measurement bases for the four homodyne detections.

In summary, we have experimentally demonstrated one-way quantum computations with continuous variables. All

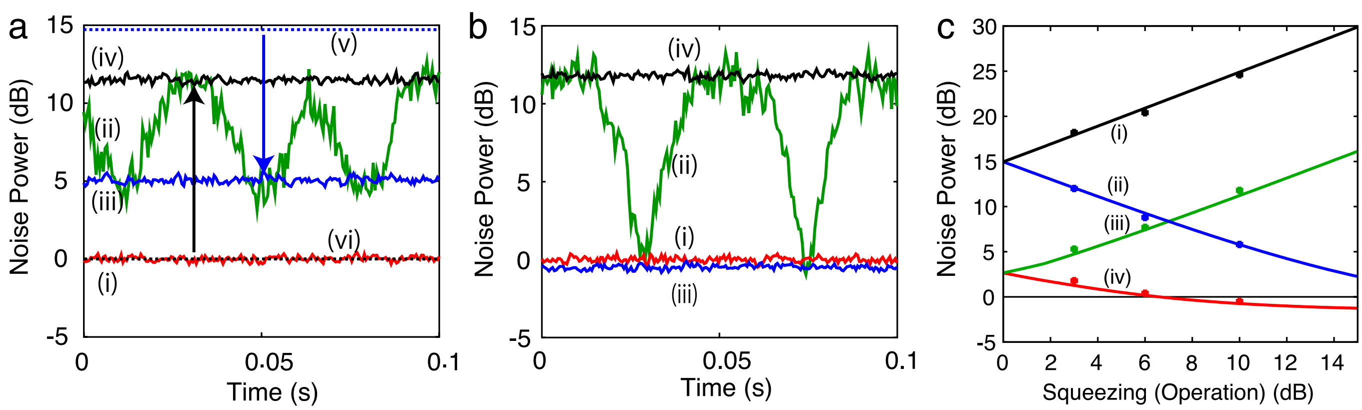

FIG. 3 (color online). Squeezing operations; (a),(b) 10 dB x-squeezing operation with an x-coherent input (a) and a vacuum input (b). Trace (i): shot noise level, trace (ii): phase scan of the output state, trace (iii): measurement of x, trace (iv): measurement of p, dotted line (v): x of the input, and dotted line (vi): p of the input. The measurement settings are the same as in Fig. 2. Traces (i), (iii), and (iv) are averaged 20 times. (c) experimental results (dots) and theoretical calculation (solid curves) of 3, 6, and 10 dB x-squeezing operations. Traces (i) and (ii) correspond to a p measurement with p-coherent input, and x measurement with x-coherent input, respectively, traces (iii) and (iv) correspond to a p measurement with vacuum input, and x measurement with vacuum input, respectively. Each data point has an error of about $\pm$0.2 dB.

operations were perfectly controllable through appropriate choice of measurement bases for the homodyne detections, and implemented in a fully unconditional fashion. In our scheme, arbitrary linear one-mode transformations can be applied to arbitrary input states coming independently from the outside. An extension to multimode transformations, though not demonstrated here, is also possible by similar means [22]. The accuracy of our one-way quantum computations only depends on the squeezing levels used to create the resource cluster state. Although in our experiment squeezing levels were sufficient to verify the nonclassical nature of the output states, even higher levels of squeezing, as reported recently [31,32], may lead to increased accuracies and one-way quantum computations of potentially larger size in the near future. In order to achieve quantum operations other than linear unitary mode transformations, nonlinear measurements besides homodyne detections would be required, or, alternatively, additional non-Gaussian ancilla states. However, the demonstration of the experimental capability of implementing an arbitrary linear single-mode transformation through continuous-variable cluster states, as presented here, represents a crucial step toward universal one-way quantum computation.

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